TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

2010-2011

Q 1. What do you mean by equilibrium

In economic sense, equilibrium refers to a state or situation in which opposite economic forces, e.g., demand and supply, are in balance and there is no in-built tendency to deviate from this position.

At macro level, an economy is said to be in equilibrium when aggregate demand equals aggregate supply. Aggregate demand is the sum of demands for all consumer and capital goods and services, given the aggregate demand for money. Aggregate supply is the sum of the supply of all consumer and capital goods and services, given the aggregate supply of money.

As long as equilibrium is not disturbed by internal or external disequilibrating factors, the economy remains in equilibrium.

Q 2. Explain the meaning of optimization.

Optimization is an act, process, or methodology of making something (such as a design, system, or decision) as fully perfect, functional, or effective as possible.

Optimization in economics means:-

Optimization is mainly concerned with finding maximum and minimum points, also known as

optimum points of a function. Applications include finding optimum values for functions such as profit, cost, revenue, production and utility. These functions which are to be maximized or minimized are called objective function.

Examples

Consumers maximize utility by purchasing an optimal combination of goods

Firms maximize profit by producing and selling an optimal quantity of goods

Firms minimize their cost of production by using an optimal combination of inputs

Q 3. What is rectangular coordinates ?

The rectangular coordinate system consists of two real number lines that intersect at a right angle. The horizontal number line is called the x x-axis, and the vertical number line is called the y y-axis. These two number lines define a flat surface called a plane, and each point on this plane is associated with an ordered pair of real numbers (x,y).

The first number is called the x x-**coordinate**, and the second number is called the y y-**coordinate**.

The intersection of the two axes is known as the origin , which corresponds to the point $(0,0)$(0,0).
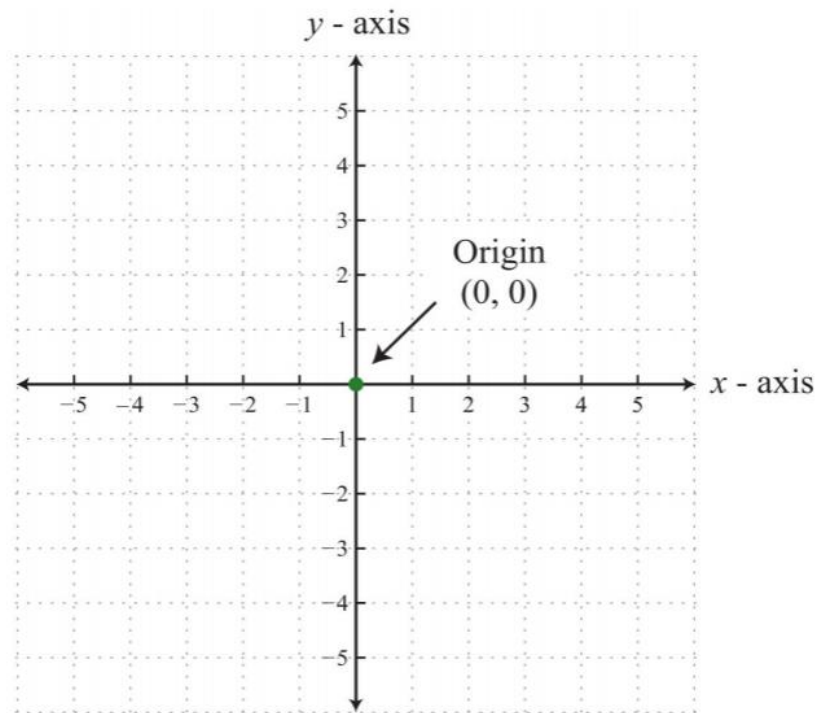
Q 4 Point out the difference between stable and unstable equilibrium

Stable equilibrium

State equilibrium is that type of equilibrium where any disturbance in the equilibrium situation is self adjusting so the old equilibrium position is restored. In other words of Marshall 'When the demand price is equal the supply price, the amount produce has so tendency either to be increased or decreased, it is an equilibrium. The stable equilibrium can be explained with the help of following diagram.

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(ECB- 201 ,2021-22)
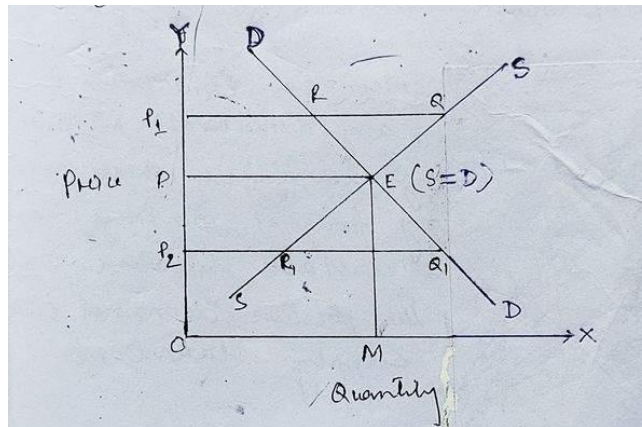
In the above diagram we have shown stable equilibrium positions. Original equilibrium point is **E** where demand and supply are equal at **OP** price level.

If price is increased to **OP1** then supply will be more than demand **(**P1Q>P1R**)**. As a result the price level will rise to **OP** again.

Further, when price falls to **OP2** then demand will be more than supply (P2Q1>P2R1). As a result the price level will rise to **OP** again.

Thus, if we disturb the demand supply interaction model through price raising and price reducing then the system will be restored itself in the equilibrium position where **D = S**. This is called **stable equilibrium.**

Unstable equilibrium

Equilibrium is unstable when any disturbance in equilibrium situation brings in forces which more the system away from it, never to be restored. This unstable situation can be explained by the positive demand curve and negative supply curve.

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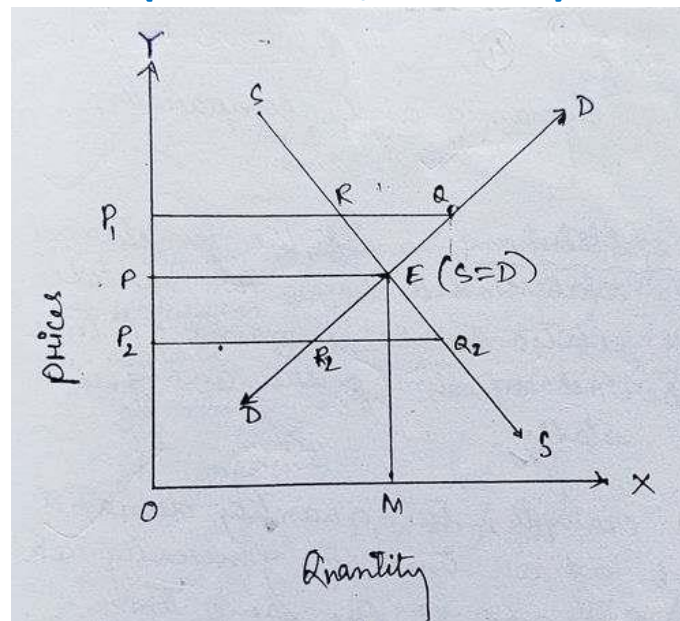
In the above figure **DD** is the demand curve for Griffin goods and SS is the supply curve under increasing cost condition. Both the curve intersects each other at point E where demand is equal supply at **OM** level of quantity.

Suppose, price increase to OP1. Here demand is more than supply (P1Q>P1R). As a result the price level will increase further and it will never restore at point **E**.

Similarly, at point OP2, the supply is more than demand (P2Q2>P2Q2). As a result the price level will decrease further and it will never restore at point E.

Q 5 What is total revenue ?

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

Total revenue

the aggregate revenue obtained by a **FIRM** from the sale of a particular quantity of output, equal to price times quantity. Under conditions of **PERFECT COMPETITION**, the firm faces a horizontal **DEMAND CURVE** at the going market price.
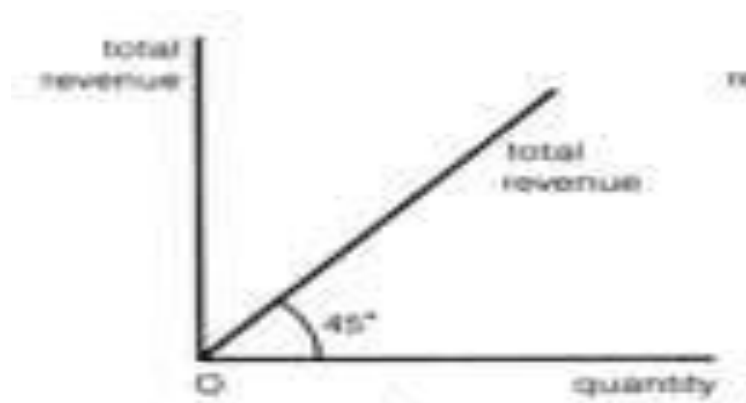
It is important to note that revenue is distinct from **earnings** or **profits**, which takes **expenses** into account. Obviously, however, high total revenue is desirable for any company.

Q 6 Define Elasticity of Demand. Explain types of elasticity giving their formulas and examples. [15 marks]**

Elasticity is a concept in economics that talks about the effect of change in one economic variable on the other.

Elasticity of Demand, on the other hand, specifically measures the effect of change in an economic variable on the quantity demanded of a product. There are several factors that affect the quantity demanded

for a product such as the income levels of people, price of the product, price of other products in the segment, and various others.

"The elasticity (or responsiveness) of demand in a market is great or small according as the amount demanded increases much or little for a given fall in price, and diminishes much or little for a given rise in price".

– Alfred Marshall, British Economist

Price elasticity of demand ⋮   Overview   F

Formula

$$e_{(p)} = \frac{dQ/Q}{dP/P}$$

$e_{(p)}$ = price elasticity
Q = quantity of the demanded good
P = price of the demanded good

Factors affecting Demand Elasticity

Three main factors affect a good's price elasticity of demand.

1. <u>Availability of substitutes</u>

In general, we can say that the more good substitutes are there, the more elastic demand will be. This can be understood by an example. Suppose a coffee seller company increases the price for its cup of coffee by $1. The consumers are likely to switch to another company or they may even replace their cup of coffee with a cup of strong tea. This means that the cup of coffee is an elastic good as a small increase in the price is resulting in a large decrease in the demand.

2. <u>Necessity</u>

This is not a mystery at all. We all need a few things for survival and we can not give up on them. These products that we require for survival are termed as necessity products. For example, rice grains. A large part of the Indian population is a daily consumer of rice grains.

So, even if the prices go higher the consumption won't decrease drastically and the demand will almost remain the same. This makes the good inelastic.

3. <u>Time</u>

The third influential factor is time. We consume some goods as we are addicted to them. Two of the most popular examples are alcohol and tobacco. We will understand the role of time with an example.

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

Suppose the government increases the taxes on tobacco which leads to an increase in the prices. So, a person addicted to smoking won't stop buying cigarettes. This makes the product inelastic. However, if the prices go on increasing and the person now can not afford to spend extra on those cigarettes, he or she may get rid of the habit. This makes the price elasticity of cigarettes for that consumer elastic in the long run.

3 Types of Elasticity of Demand

On the basis of different factors affecting the quantity demanded for a product, elasticity of demand is categorized into mainly three categories: **Price Elasticity of Demand (PED), Cross Elasticity of Demand (XED), and Income Elasticity of Demand (YED)**.

1. Price Elasticity of Demand (PED)

Any change in the price of a commodity, whether it's a decrease or increase, affects the quantity demanded for a product. For example, when there is a rise in the prices of ceiling fans, the quantity demanded goes down.
 This measure of responsiveness of quantity demanded when there is a change in price is termed as the Price Elasticity of Demand (PED).
 The mathematical formula given to calculate the Price Elasticity of Demand is:

PED = % Change in Quantity Demanded % / Change in Price

The result obtained from this formula determines the intensity of the effect of price change on the quantity demanded for a commodity.

2. <u>Income Elasticity of Demand (YED)</u>

The income levels of consumers play an important role in the quantity demanded for a product. This can be understood by looking at the difference in goods sold in the rural markets versus the goods sold in metro cities.

The Income Elasticity of Demand, also represented by YED, refers to the sensitivity of quantity demanded for a certain good to a change in real income (the income earned by an individual after accounting for inflation) of the consumers who buy this good, keeping all other things constant.

The formula given to calculate the Income Elasticity of Demand is given as:

YED = *% Change in Quantity Demanded% / Change in Income*

The result obtained from this formula helps to determine whether a good is a necessity good or a luxury good.

3. <u>Cross Elasticity of Demand (XED)</u>

In a market where there is an oligopoly, multiple players compete. Thus, the quantity demanded for a product does not only depend on itself but rather, there is an effect even when prices of other goods change.

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

Cross Elasticity of Demand, also represented as XED, is an economic concept that measures the sensitiveness of quantity demanded of one good (X) when there is a change in the price of another good (Y), and that's why it is also referred to as Cross-Price Elasticity of Demand.

The formula given to calculate the Cross Elasticity of Demand is given as:

XED = *(% Change in Quantity Demanded for one good (X)%) / (Change in Price of another Good (Y))*

The result obtained for a substitute good would always come out to be positive as whenever there is a rise in the price of a good, the demand for its substitute rises. Whereas, the result will be negative for a complementary good.

For example, if the price of coffee increases, the quantity demanded for coffee stir sticks drops as consumers are drinking less coffee and need to purchase fewer sticks. In the formula, the numerator (quantity demanded of stir sticks) is negative and the denominator (the price of coffee) is positive. This results in a negative cross elasticity.

These three types of Elasticity of Demand measure the sensitivity of quantity demanded to a change in the price of the good, income of consumers buying the good, and the price of another good.

Apart from these three types, we have some other types of Elasticity of Demand which we would look at now.

The effect of change in economic variables is not always the same on the quantity demanded for a product.

The demand for a product can be elastic, inelastic, or unitary, depending on the rate of change in the demand with respect to the change in the price of a product.

On the basis of the amount of fluctuation shown in the quantity demanded of a good, it is termed as **'elastic'**, **'inelastic'**, and **'unitary'.**

- An **elastic demand** is one that shows a larger fluctuation in the quantity demanded of a product, in response to even a little change in another economic variable. For example, if there is a hike of $0.5 in the price of a cup of coffee, there are very high chances of a steep decline in the quantity demanded.
- An **inelastic demand** is one that shows a very little fluctuation in the quantity demanded with respect to a change in another economic variable. An example of this can be petrol or diesel.
- **Unitary elasticity** is one in which the fluctuation in one variable and quantity demanded is equal

Types of Price Elasticity of Demand: Condition

NUMERICAL VALUE	TYPE OF PRICE ELASTICITY OF DEMAND	CONDITION
$=\infty$	Perfectly elastic demand	Greater change in demand in response to percentage or smaller change in the price.
$=0$	Perfectly inelastic demand	No change in demand in response to percentage or smaller change in the price.
>1	Relatively elastic demand	A change in demand is greater than the change in price.
<1	Relatively inelastic demand	A change in demand is less than the change in price.
$=1$	Unitary elastic demand	A change in demand is equivalent to change in price.

2013-2014

Q 7 What is Parabola ?

a parabola as the set of all points (x, y) in a plane that are equidistant from a fixed line, the directrix, and a fixed point, the focus, not on the line. The vertex is the midpoint between the focus and the directrix. The axis is the line through the focus and perpendicular to the directrix.
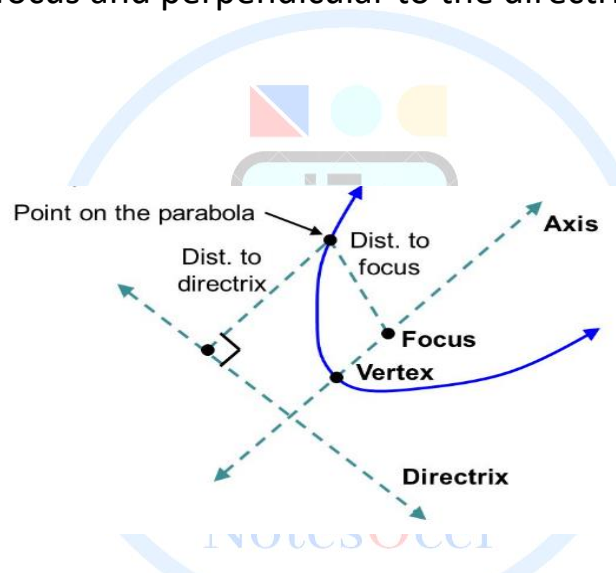
Equation of Parabola

If the directrix is parallel to the y-axis then the equation of a parabola is

$y^2 = 4ax$

where a:- Distance of focus from the origin {0,0}

Q 8 "Statics is a timeless concept ,whereas dynamics involves times" explain fully this statement

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

Static Economics

In Economics, the concept of static refers to a situation where there is a movement. But this movement is continuous ,certain ,regular and constant. Static economics does not deal with unexpected changes . it studies only the expected economic activities According to prof. Harrod , "An economy in which rate of output is constant is called static ."

According to

Schumpeter, a static economy refers to "an economic process that merely reproduces itself." When an economy is studied under static conditions, it is called static analysis. For static analysis, a static model is used. A model of an abstract economy is created by a "rigorous formulation of conditions [assumptions] under which it is possible to make generalisations about the factors determining economic equilibrium."

For example, a table placed in a room, a book lying on the table, and a car parked on the road is in the state of rest or motionlessness. A static economy means an economy or in which normal activities go on but there is no change in the size of the economy or in the level of national output, stock of capital, prices and employment.

For instance , we say market is in equilibrium when D=S ,which is graphically represented by the intersection point of demand and supply curve. This is static analysis since we just see the pictures at one point of time only.

Fig. 1.2 Economic Statics : Equilibrium of a competitive market at a point of time is static economics
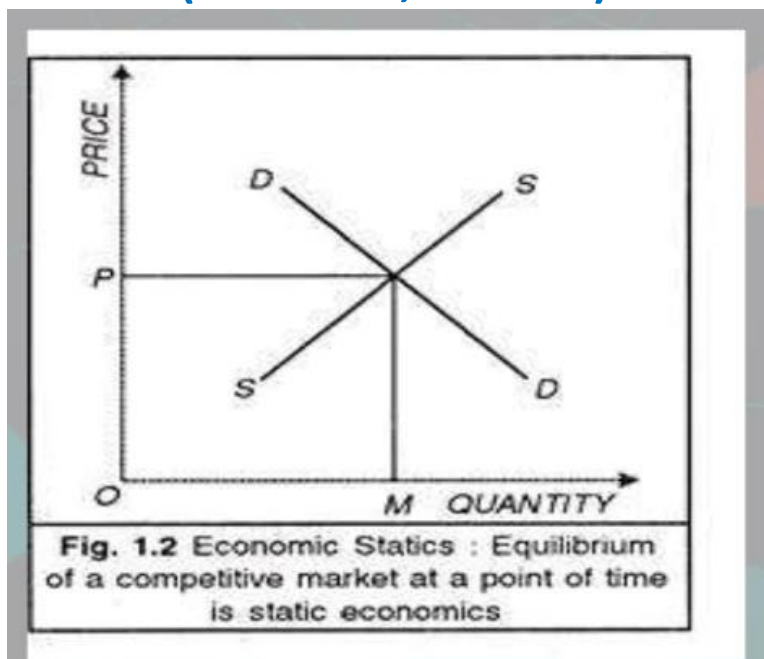
Features of static economics :-

1. No change in population and it's composition
2. No change in quantity of capital
3. No change in techniques of production
4. No change in habits ,workin and organization of industrial units.

Limitations:-

1. Constancy of variable :- Prof. Clark and stigler have presumed many economic variables as constant. They are population ,quantity of capital, natural resources , techniques of production etc. We know that these economic factors changes in reality.So static economic analysis is far from reality.
2. It ignores time element :-It studies a timeless economy ,but in reality many changes occurs with passage of time.
3. It does not explain the path of equilibrium :- Static analysis explains only the final state of equilibrium, it does not tells how this new equilibrium has been reached.

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

Dynamic Economics

In contrast to static approach, dynamic approach is adopted to study an economy in motion. When a macroeconomic phenomenon is analysed under changing or dynamic conditions, it is called dynamic analysis. Dynamic analysis is adopted to study an economy under dynamic conditions. In a dynamic economy, the economic factors and forces keep changing.

We have know that there is a movement in statics also but this movement is regular and expected. While dynamics refers to that movement which is uncertain, unexpected and irregular.
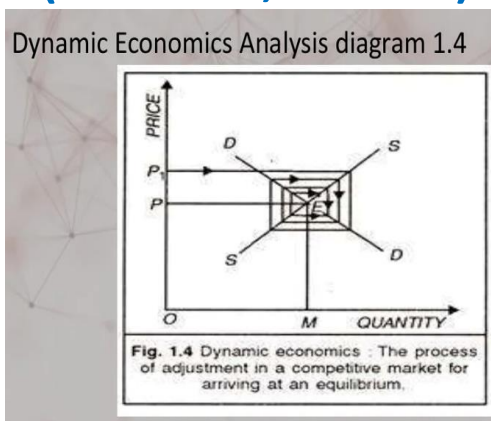
According to Prof. Harrod, "Dynamics economics is the study of an economy in which rates of output are changing."

In dynamic we focus on the change of time and how the equilibrium change with time. It is same as watching the movie you can see how the image animate and move. Dynamic analysis allow us to see the path of variable how the variable change with time. It help us to see whether the equilibrium will reach or not.

The concept of dynamic is nearer to reality.In dynamic economics we study the economic variables like consumption function ,income and investment in a dynamic state.

In dynamic economics we also study the path of change or the movement towards equilibrium. This path can be explained with the help of the diagram given below which relates to price determination in the market.

Dynamic Economics Analysis diagram 1.4

Fig. 1.4 Dynamic economics : The process of adjustment in a competitive market for arriving at an equilibrium.

Limitations:-

Dynamics economics analysis is a complex approach for the study of economic variables because it is based on time element. To find solutions of various problems, we have to make use of mathematics and economics which is beyond the understanding of common man.

Distinction between Economic Statics and Dynamics

The distinctive features of static and dynamic analyses can be summarised as follows.

(i) Economic statics is an abstraction from reality whereas economic dynamics is the study of the real world.

(ii) All the variables in a static analysis are undated in the sense that they are taken at a point or unit of time whereas in dynamic analysis, all variables are dated, i.e., their movement on time scale is known.

(iii) Economic statics is a timeless analysis whereas in economic dynamics, time is used as one of the variables because time works as a determinant of other variables. For example, national income of a country in time t depends on its value in time t–1.

(iv) In static analysis, fundamental economic conditions are assumed to be given and known, but in a dynamic analysis, they continue to change over time.

(v) Dynamic analysis has predictive power which static analysis does not have, though comparative statics can be used for the purpose.

Q 9 Discuss the relationship between average revenue ,Marginal revenue and Elasticity of demand ?

Average revenue is referred to as the revenue that is earned per unit of output. In other words, it is the revenue that is obtained by the seller on selling each unit of the commodity.Average revenue of a business is obtained by dividing the total revenue with the total output.The average revenue is similar to the price if a seller sells two units of the same product at the same price. However, the average revenue varies if the two products are sold at two different prices.

Average revenue helps in estimating the profit of a business, as the profit is calculated by subtracting the average revenue from the average cost.

A market structure determines the relationship between the average revenue and the quantity of goods produced. In a perfectly competitive firm, the average revenue is equal to the price and the marginal revenue.

However, in monopolistic or oligopolistic firms, the average revenue is always higher than the marginal revenue.

The AR curve of a firm is also the same thing of Demand curve for the consumer's demand of firm products.

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The mathematical formula for calculating average revenue is given as follows:

AR = TR/Q

Where,

AR = Average revenue

TR = Total revenue

Q = Output

This concept is about the average revenue formula. It is a very important concept for determining the profit of a business.

Marginal revenue is referred to as the revenue that is earned from the sale of an additional product or unit. It is the revenue that the company generates when there is a sale of an additional unit.It is a microeconomic term that has many applications in accounting. Marginal revenue formula is the formula to calculate marginal revenue.

It is used by the management to analyse the customer demand, plan the production schedules, and set product prices. If the customer demand is not appropriately judged, then it results in the loss of sales and excess production, which, in turn, results in increased manufacturing costs

Marginal revenue remains constant till a certain level of output is achieved. It then slows down with increasing output by following the law of diminishing returns.

By setting the price for a product, manufacturers can alter its demand. For example, if the price of a product is raised, then it reduces its demand and also reduces the need for its manufacturing.

It might also lead to higher revenue per product. However, the downside is that it can cause the buyers to shift to competitors which can result in the business losing out on more sales.

The marginal revenue formula is as follows:

Marginal revenue = Change in total revenue/Change in quantity

Or, **MR = ΔTR/Δq**

Where,

ΔTR = Change in total revenue

Δq = Change in quantity

Elasticity of demand :- Refer Q 6

Relationship between Average Revenue, Marginal Revenue and Price Elasticity of Demand

Q 9) Discuss the relationship between average, revenue, marginal revenue and elasticity of demand.

A9.   Prerequisite Req. → HLAtuja ; Principles of Microeconomics ; Elasticity of Demand and Supply (Pg :- 161- 163))
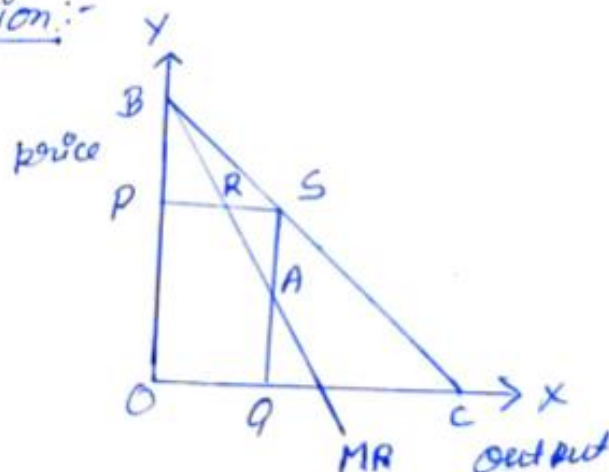
There is a crucial relationship between AR, MR and elasticity of demand. Which is used extensively in theory of pricing. The relationship is expressed in formula as.

$$AR = MR \left(\frac{e}{e-1} \right)$$

or,

$$MR = AR \left(1 - \frac{1}{e} \right) \quad \text{(taking reciprocals both sides)}$$

derivation:-

A point 'S', elasticity (e) is

$$\frac{SC}{SB} \quad \left(\because e_p = \frac{\text{lower segment of slope}}{\text{upper } \quad '' \quad '' \quad ''} \right)$$

Now, $\dfrac{SC}{SB} = \dfrac{SQ}{PB}$ $\quad (\because \triangle SQC \sim \triangle BPS \text{ by AA similarity})$

Therefore, $e = \dfrac{SQ}{PB} = \dfrac{SQ}{SA}$ $\quad \left(\because \triangle RBP \cong \triangle RAS \text{ by ASA congruency as } PR = RS \right)$

$$= \frac{SQ}{SQ - AQ}$$

But, AQ is marginal revenue & SQ is AR at OQ level of output.

$$\therefore \quad e = \frac{AR}{AR - MR}$$

$$\therefore \quad AR = e(AR - MR)$$

$$AR(1 - e) = -eMR$$

$$eMR = (e - 1)AR$$

$$MR = AR\left(1 - \frac{1}{e}\right)$$

The relation between AR, MR and elasticity of price demand (e) can be written as:-

$$\boxed{e = \left(\frac{AR}{AR - MR}\right), \quad MR = AR\left(\frac{e-1}{e}\right) \text{ and } AR = MR\left(\frac{e}{e-1}\right)}$$

where , e = price elasticity of demand
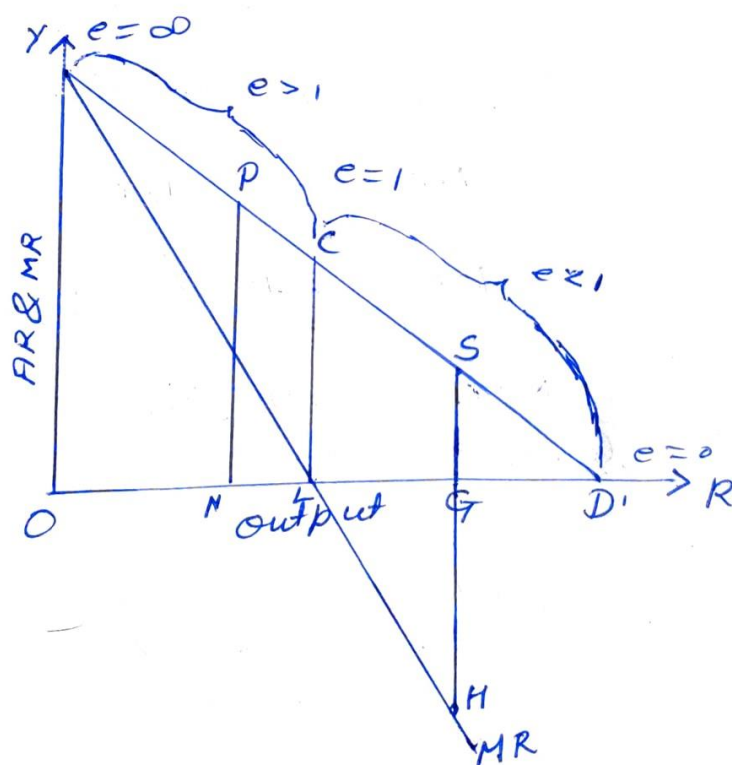
AR = Average revenue ; MR = Marginal Revenue

With the help of the above formula, we can find MR at level of output from AR at same output & we know ep on the AR curve. Like, If ep = 1 at given level of output, then MR = 0.

$$\therefore \quad M = A \left(1 - 1/e\right)$$

$$= A (1-1) = A \times 0 = 0$$

It will be seen from figure belows that corresponding to the middle point C on the AR curve DD', where e = 1 , the MR = 0

To sum up, marginal revenue is always positive at any point or output where the elasticity of the average revenue curve is greater than one and marginal revenue is always negative where the elasticity of average revenue curve is less than one and marginal revenue is zero corresponding to unit elasticity at the average revenue curve.

Q 10 Write short notes on

(i) Scarcity & Choice

(ii) Fundamentals problems of an economy

Scarcity

The wants of human beings are limitless and resources to fulfill them are limited. It is a fact that the total quantity of products that can be produced by applying the productive resources of an economy is insufficient to satisfy all the needs and wants of the people. When the wants of people exceed their resources then it is known as scarcity. Scarcity is the fundamental problem of every society/nation. As a consumer, all the time we people always face scarcity of something like scarcity of income, scarcity of products, etc. Therefore, scarcity is the situation in which people have limited resources to satisfy all their desires.

economic scarcity refers to the basic fact of life that there exists only a limited amount of human and non-human resources and due to which people can produce only a limited amount of economic goods. Therefore, it is not possible for every economy to produce various

types of goods in unlimited quantity as the economy has limited resources only.

Choice

Since the resources available to satisfy people's wants are limited in supply, so people cannot fulfill all their wants and they must make choice. It means people must economize the resources. That is every economy must make decisions about the best use of available resources in the consumption of various goods and services. So, preferring the most satisfying alternative amongst available is known as choice.

Scarcity gives birth to the choice or to the problem of choice. The limitation of resources forces economic participants to choose. For an individual, scarcity pushes his/her to choose only those goods which give more satisfaction. The producer is always in the compulsion to choose to produce higher profit ensuring products.

The problem of scarcity and choice for **economic agents** applies in the following ways.

Consumers: *Due to scarcity they always must choose those goods which give them the most utility.*

Producer: *Producers due to limited budget need to choose those products which give them maximum profit.*

Government: *The government also faces the limitation of resources and it must choose those programs and projects which maximize people's welfare.*

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

Alternative use of scarce resources creates problems of choice. Thus, scarcity and choice or selection are the major economic issues and economics aims to solve such problems faced by its entities.

Fundamental problems of an economy

An economic problem generally means the problem of making choices that occurs because of the scarcity of resources. It arises because people have unlimited desires but the means to satisfy that desire is limited. Therefore, satisfying all human needs is difficult with limited means.

The three Central Problems of an Economy are?

- What to Produce and in What Quantity?

- How to Produce?

- For Whom to Produce?

(A) What to produce?

- A country cannot produce all goods because it has limited resources.
- It has to make a choice between different goods and services.
- Every economy has to decide what goods and services should be produced.

- Example: If a farmer has a single piece of agricultural land, then he has to make a choice between two goods, i.e., whether to grow rice or wheat.
- Similarly, our government has to decide where to allocate funds, for the production of defence goods or consumer goods, and if both, then in what proportion.

(B) How to produce?

- This problem refers to the choice of technique of production. It arises when there is an availability of more than one way to produce goods and services.
- There are mainly two techniques of production. These are:
  - Labour intensive technique (greater use of labour)
  - Capital intensive technique (greater use of machines)
- Labour intensive technique promotes employment whereas capital intensive technique promotes efficiency and growth.

(C) For whom to produce?

- The society cannot satisfy all the wants of all the people. Therefore, it has to decide who should get how much of the total output of goods and services.
- Society has to make choice of whether luxury goods or normal goods have to be produced. This distribution or proportion directly relates to the purchasing power of the economy.

Q 11 Distinguish between partial and general equilibrium approach to economic analysis. What are the advantages and limitations of general equilibrium [15]

Meaning of Equilibrium

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See Q 1

Approaches to Equilibrium in Economics

In economics, the study of equilibrium can be done in two different approaches based on the considered subject of equilibrium. If we take a particular part of the economy considering no change in other parts or ignoring the change in other parts and their effects then it is called partial equilibrium analysis. On the other hand, if we consider all the markets of the entire economy in a framework then it is called general equilibrium analysis. Partial and general equilibrium has its own important contribution to economic analysis.

Partial Equilibrium

The concept of partial equilibrium is introduced by **Alfred Marshall**.
When we study the behavior of individual decision-making units and the working of individual markets for commodities and inputs under various market structures it is a case of partial equilibrium analysis.

It can be said that the partial equilibrium approach deals with each market independently without considering the effects of changes in other markets on the concerned market. The advantage of partial equilibrium is that it provides room for the

analysts to focus on one thing at a time and thus avoid the puzzlement that can arise the entire economy is to be considered.

The partial equilibrium analysis does not observe how the various individual parts fit together to form an integrated economic system. This task is not here and is left to the duty of general equilibrium analysis.

Partial equilibrium studies in microeconomic analysis. In microeconomics, there is a study of the market for a particular commodity or a particular factor input. The price of a commodity is determined when the demand for that commodity is equal to its supply.

For example, the wage rate is determined when the demand for labor is equal to its supply. The interest rate is determined when the demand for savings is equal to its supply. In all these markets the equilibrium is attained at the point where the demand and supply are equal and no one has the inducement to disturb the position. This analysis is called partial equilibrium analysis. It focuses on the determination of equilibrium prices and quantities in a market ignoring the effect from other markets.

Characteristics of the Partial Equilibrium Approach

It studies the behavior of economic participants

In partial equilibrium analysis, the behavior of individual decision-making units and individual markets is considered in separation. It examines how an individual maximizes his satisfaction subject to his income constraint. How a firm minimizes its cost of

production, how a firm maximizes profits under various market conditions how the prices and employment of each type of input are determined.

For example, the rise in price of petrol following imposition of a tax on it would cause little effect on the prices of goods such as wrist watches, drapers, bowling balls, and in turn there would be negligible feedback effect of changes in prices of these goods on the demand and price of petrol.

If prices of petrol and of only these commodities are to be considered and since there are little repercussions of changes in prices of petrol on these other commodities, the use of partial equilibrium analysis of price determination of petrol would be quite reasonable.

However, when market for automobiles is considered, the rise in price of petrol would have an important effect on their demand and price. Therefore, the assumption of partial equilibrium analysis that prices of automobiles would remain constant, when the price of petrol changes would be seriously wrong.

Relationships with other markets are ignored.
Partial equilibrium analysis studies the behavior of individual economic segments in isolation and ignores their interdependence.

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

Limitations

1.It delas with only one economic factor

2.It does not deal with entire economy

3.It is based on ceteris paribus

4.Its assumptions are unrealistic and hence unsuitable to study world phenomenon.

5.Its analysis is incomplete. Its studies only primary effects and does not consider secondary effects.

6.Seggregating individual behaviour from the rest of economy is unrealistic.

Use of Partial Equilibrium Analysis

The partial equilibrium is considered a useful model on the following grounds

For example effect of a labor strike in the steel industry. Each market in partial equilibrium analysis is regarded as independent of the other.

Partial equilibrium analysis studies these initial effects or impact effects only. For example, the impact of a labor strike in the steel industry first is seen in steel production. This can be termed as first-order impact or impact effects. Later on, there will be spillover effects on the entire economy and this is termed the second and higher-order impacts. The higher-order impacts will be

seen in the next instantaneous industry that usages still as input and ultimately in the entire economy.

General Equilibrium Analysis

The general equilibrium analysis considers the fact of interdependence among different economic units. Interdependence in the economy makes partial equilibrium analysis very simple because demand and supply in one market depend on prices determining other markets. This concept was introduced by **Leon Walras**.

General equilibrium is an extensive study of several economic variables their interrelation and interdependence for analyzing the working of the economic system as a whole.

Everything depends on everything else is a fundamental characteristic of the modern economy. It means interdependence among its part parts is an essential feature of modern economies. This feature is recognized and applied by general equilibrium in its analysis. As a result, it is considered that the markets of the commodities and all predictive factors are interrelated and thus the prices in all markets are simultaneously determined.

For example, consumers' demand for various goods and services depends on their tastes and incomes. Consumer incomes also depend on the demand and supply of the various inputs. The demand for factor inputs by firms depends not only on the state of technology but also on the demand for the final goods they produce. The demand for these goods depends on consumer incomes which depend on the demand for the factors of production. Thus the change in one market affects other markets which in turn affects the original market.

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION
(ECB- 201 ,2021-22)

The disturbance in one market affects the entire economic system and the general equilibrium analysis concerns itself with the changes caused in the whole economy.

Here any change in one market affects other markets like the spillover effect and is affected by other markets called the feedback effects.

Assumptions:

The general equilibrium analysis is based on the following assumptions:

(1) There is perfect competition both in the commodity and factor markets.

(2) Tastes and habits of consumers are given and constant.

(3) Incomes of consumers are given and constant.

(4) Factors of production are perfectly mobile between different occupations and places.

(5) There are constant returns to scale.

(6) All firms operate under identical cost conditions.

(7) All units of a productive service are homogeneous.

(8) There are no changes in the techniques of production.

(9) There is full employment of labour and other resources.

Use of General Equilibrium Analysis

General equilibrium analysis has two basic purposes

It ensures room for examining and analyzing the economic system as a whole.

It provides a logical and systematic approach to studying higher-order effects, ripples effect, and repercussion effects of an economic change.

General equilibrium is thus defined as a situation in which every market and every decision-making unit are simultaneously in equilibrium. Any economy is said to be in the state of general equilibrium if there prevails a set of prices that can link demand and supply and produce equilibria in every product and factor market that are mutually consistent.

Limitations:

The general equilibrium analysis of the economy has several limitations:

1. It is based on a number of unrealistic assumptions which are contrary to the actual conditions prevailing in the world. Perfect competition, the very basis of this analysis, is a myth.

2. It is a static analysis. All consumers and producers in this analysis consume and produce the same products day in and day out without any time-lag. Their tastes, preferences, and aims are the same, and their economic decisions are in perfect harmony with each other.

In reality, nothing of this sort happens. Producers and consumers never act and think alike. Changes are taking place continuously in tastes and preferences. There are no constant returns to scale and no two factor services are homogeneous. Thus cost conditions differ from producer to producer. Since the given conditions are continuously changing, the movement towards general equilibrium is ever thwarted and its attainment has ever remained a wishful ideal.

3. Prof. Stigler regards general equilibrium as a misnomer. According to him, "No economic analysis has ever been general in the sense that it considered equilibrium studies as more inclusive than partial equilibrium studies, never that they are complete. Moreover, the more general the analysis, the less specific its content must necessarily be."

Comparison between Partial and General Equilibrium

SL.No	Partial Equilibrium	General Equilibrium
1.	Partial equilibrium analysis focuses on the market in which the tax is imposed.	General equilibrium analysis looks at various markets.
2.	Partial Equilibrium analysis looks only at the effects on one generation.	General equilibrium analysis include intergenerational redistribution.
3.	Partial equilibrium analysis looks only at prices.	General equilibrium analysis allows both quantities and prices to adjust.
4.	Partial equilibrium analysis is short term analysis.	General equilibrium analysis takes long term adjustments into account.
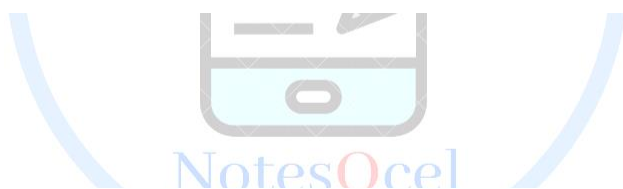
Q 12 Explain Price elasticity of demand. Explain different methods of measuring Elasticity of demand

See Q 6

Methods of Measuring Price Elasticity of Demand

1.Percentage or Proportion Method

The percentage or proportionate method measures the price elasticity of demand by dividing the percentage or a proportionate change in quantity demanded by the percentage or a proportionate change in the price of the product. It applies the following formula to compute the coefficient of price elasticity of demand

$$e_P = - \frac{\text{Percentage change in quantity demanded}}{\text{Percentage change in price}}$$

$$= - \frac{\%\Delta\ Qd}{\%\Delta\ P}$$

$$= - \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

Where,

e_P = Price elasticity of demand

Q = Original quantity demanded

ΔQ= Change in quantity demanded (Q_1 -Q)

P = Original price

ΔP = Change in price (P_1 -P)

While using a percentage or proportion method of measuring price elasticity of demand, its formula includes a negative sign as there is an inverse relationship between price and quantity demand of the commodity. Hence the computation of price elasticity of demand always results in a negative sign coefficient of elasticity. Similarly, the percentage method of measurement of price elasticity of demand is a unit free measurement as it only

considers the percentage change of price and the resulting percentage change in quantity demanded.

Another advantage of this method is that it gives a precise and exact measure of elasticity. A larger coefficient of elasticity says that demand is more sensitive towards changes in price and a small value of the coefficient of price elasticity indicates that the quantity demand is not that much price sensitive.

However, one of the issues of using this method is that the percentage change depends on the base or the starting point.

2.The Point Method:

Prof. Marshall devised a geometrical method for measuring elasticity at a point on the demand curve. Let RS be a straight line demand curve in Figure 11.2. If the price falls from PB(=OA) to MD(=OC). the quantity demanded increases from OB to OD. Elasticity at point P on the RS demand curve according to the formula is: $E_p = \Delta q/\Delta p \times p/q$
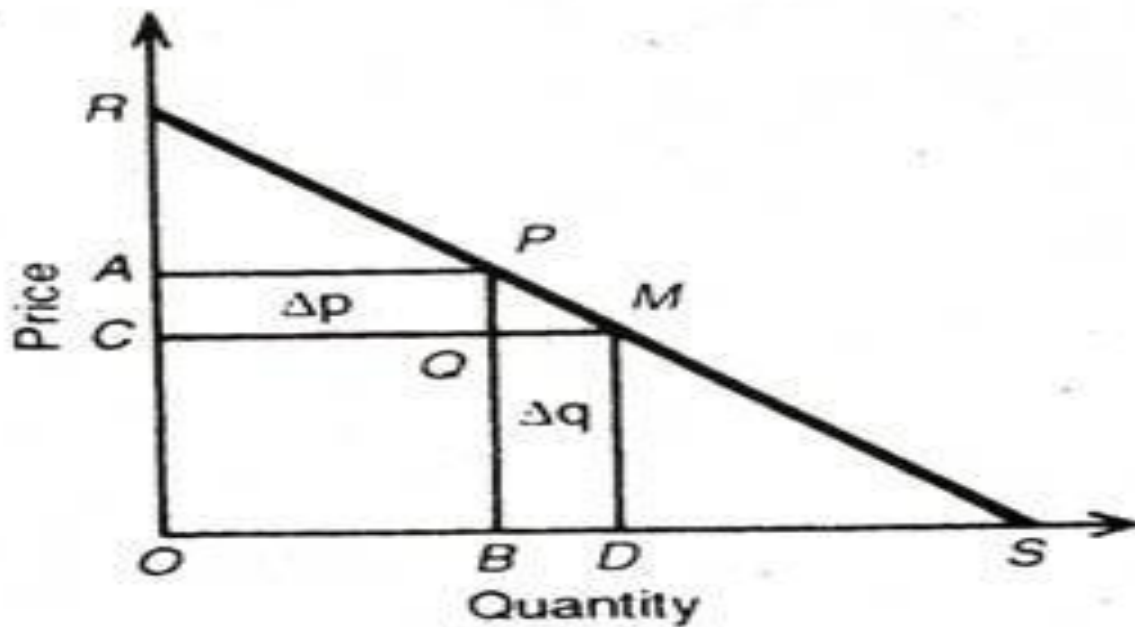
Figure 11.2

Where Δ q represents changes in quantity demanded, Δp changes in price level while p and q are initial price and quantity levels.

From Figure 11.2

Δ q = BD = QM

Δp = PQ

p = PB

q = OB

Substituting these values in the elasticity formula:

$$E_p = \frac{QM}{PQ} \times \frac{PB}{OB}$$

Moreover,

$$\frac{QM}{PQ} = \frac{BS}{PB} \quad [\angle PQM = \angle PBS \text{ being right}$$

angles and PQM and PBS are similar Δ s]

$$\therefore \frac{BS}{PB} \times \frac{PB}{OB} = \frac{BS}{OB}$$

Since ΔPBS and ΔROS are similar,

E_p at   point P $=$

$$\frac{BS}{OB} = \frac{OA}{AR} = \frac{PS}{PR} = \frac{\text{Lower Segment}}{\text{Upper Segment}}$$

With the help of the point method, it is easy to point out the elasticity at any point along a demand curve. Suppose that the straight line demand curve DC in Figure 11.3 is 6 centimetres. Five points L, M, N, P and Q are taken oh this demand curve. The elasticity of demand at each point can be known with the help of the above method. Let point N be in the middle of the demand curve. So elasticity of demand at point.
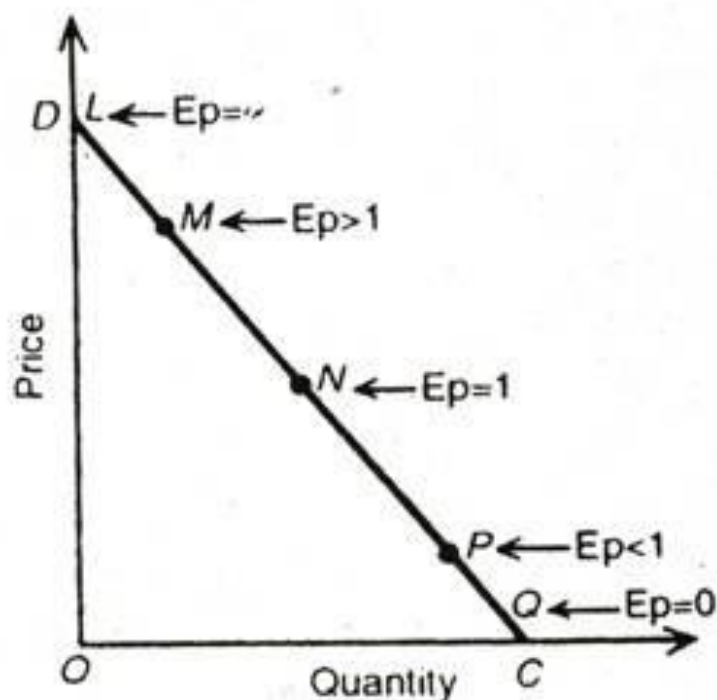
Figure 11.3

$$N = \frac{CN \text{ (Lower Segment)}}{ND \text{ (Upper Segment)}} = \frac{3}{3} = 1 \text{ (Unity)}$$

Elasticity of demand at point $M = \frac{CM}{MD} = \frac{5}{1} = 5 \text{ or} > 1$

Elasticity of demand at point $L = \frac{CL}{LD} = \frac{6}{0} = \infty \text{ (Infinity)}.$

Elasticity of demand at point $P = \frac{CP}{PD} = \frac{1}{5} = \text{ (Less than Unity)}.$

Elasticity of demand at point $Q = \frac{CQ}{OD} = \frac{0}{6} = 0 \text{ (Zero)}$

We arrive at the conclusion that at the mid-point on the demand curve the elasticity of demand is unity. Moving up the demand curve from the mid-point, elasticity becomes greater. When the demand curve touches the Y-axis, elasticity is infinity. Ipso facto, any point below the mid-point towards the X-axis will show elastic demand.

Elasticity becomes zero when the demand curve touches the X-axis.

The Arc Method:

The point method of measurement of price elasticity of demand is not accurate as we cannot get the information on very small changes in price and quantity demanded in the market. Thus, in the case of a large change in price and quantity or a time-lagged change in price, the point method is not suitable to measure the price elasticity. To avoid these defects, the arc method of measurement of price elasticity of demand is taken as an alternative method to calculate price elasticity.

Here the term ARC refers to the section or portion of a demand curve between two points. The arc method considers the average value of the initial and final values of price and quantities to measure the elasticity coefficient. Thus, it is also known as the Average Elasticity Method or Mid-Point Method. To calculate

price elasticity, first, we take the mid-point or average of prices and quantities as:

Average or Mean or Mid-point of price P= (P$_1$+P$_2$)/2
Average or Mean or Mid-point of quantity Q= (Q$_1$+Q$_2$)/2

Thus, the formula required under this method is:

$$Ep= -\frac{\dfrac{Change\ in\ Quantity\ Demanded}{Average\ Quantity\ Demanded}}{\dfrac{Change\ in\ Price}{Average\ Price}}$$

$$Ep= \frac{\Delta Q}{\Delta P} \times \frac{\frac{P_1+P_2}{2}}{\frac{Q_1+Q_2}{2}} \qquad \therefore Ep= \frac{\Delta Q}{\Delta P} \times \frac{P_1+P_2}{Q_1+Q_2}$$

Where,

ΔQ= Change in quantity demanded
ΔP= Change in price
P_1= Initial price
P_2= Final price
Q_1= Initial quantity demanded
Q_2= Final quantity demanded

The following figure and subsequent explanation show the detail regarding the measurement of price elasticity of demand under the arc method.

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION
(ECB- 201 ,2021-22)
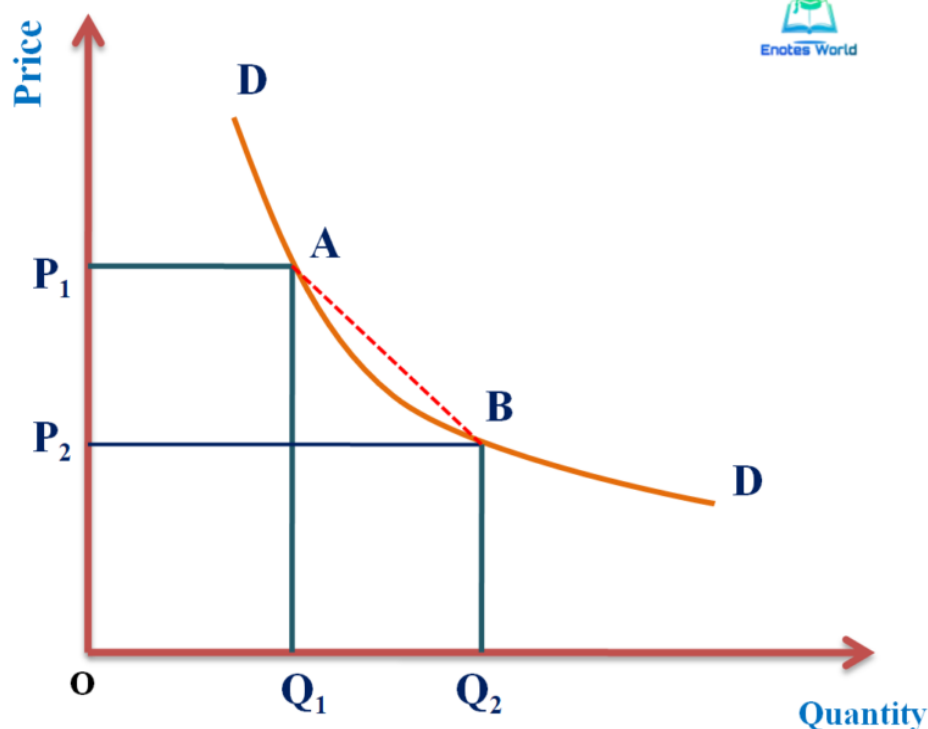
In figure DD is the demand curve and based on the change in price and quantity, e_p is calculated and which has the same value from A to B or from B to A because we take the average values while calculating the coefficient of elasticity. Thus, under such a method, the movement from A to B or movement from B to A gives the same value of e_p.

Thus, the point method of measurement of price elasticity of demand is used in the case when the changes in price and quantity are small but we use the arc method to measure the elasticity of demand when the change in price and quantity is relatively large. When the elasticity has to measure over some range or arc along a demand curve rather than at a particular point, the point elasticity does not provide the true and correct magnitude of price elasticity of demand.

The Total Outlay Method:

Marshall evolved the total outlay, total revenue or total expenditure method as a measure of elasticity. By comparing the total expenditure of a purchaser both before and after the change in price, it can be known whether his demand for a good is elastic, unity or less elastic. Total outlay is price multiplied by the quantity of a good purchased: Total Outlay = Price x Quantity Demanded.

(i) Elastic Demand:

Demand is elastic, when with the fall in price the total expenditure increases and with the rise in price the total expenditure decreases Demand is elastic ($E_p > 1$) in this case.

(ii) Unitary Elastic Demand:

When with the fall or rise in price, the total expenditure remains unchanged; the elasticity of demand is unity. $E_p = 1$.

(iii) Less Elastic Demand:

Demand is less elastic if with the fall in price the total expenditure falls and with the rise in price the total expenditure rises. This is the case of inelastic or less elastic demand, $E_p < 1$.

Table summarises these relationships:

Price	TE	E_p
Falls	Rises	>> 1
Rises	Falls	
Falls	Unchanged	= 1
Rises	Unchanged	
Falls	Falls	
Rises	Rises	<< 1
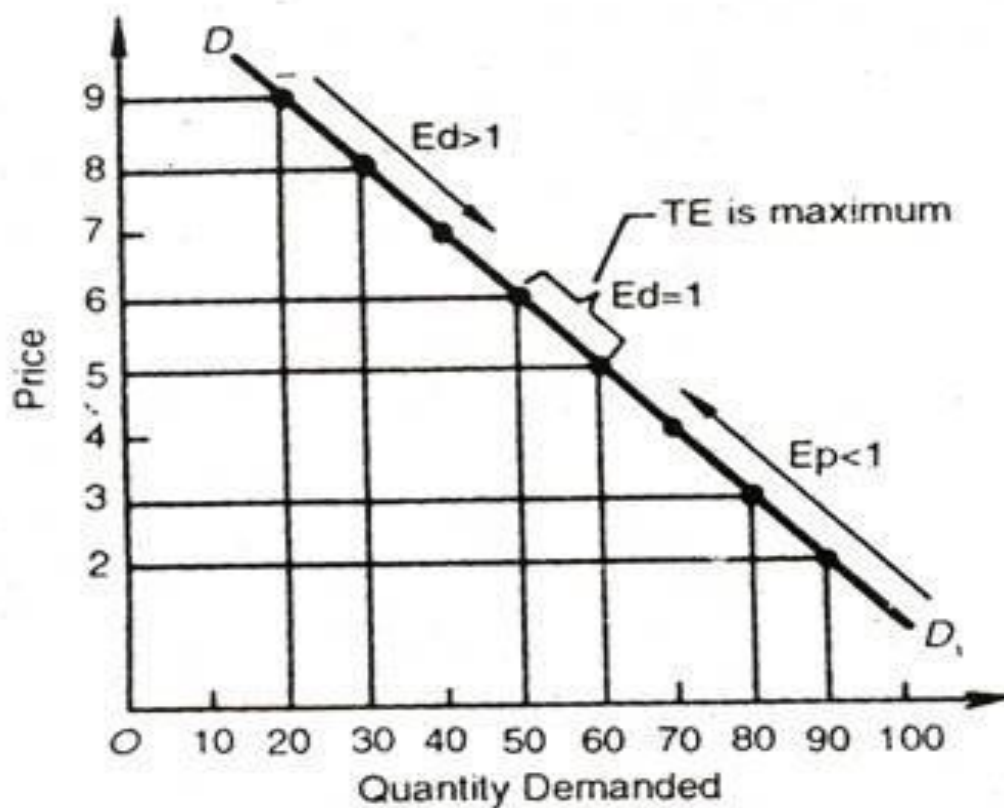
Figure 11.5

2016-2017

Q 13 Explain the relationship between average cost and marginal cost with the help of suitable diagram.**

Prerequisite req. :- HL AHUJA ;Priciples of Microeconomics [pg 399-404]

Marginal Cost
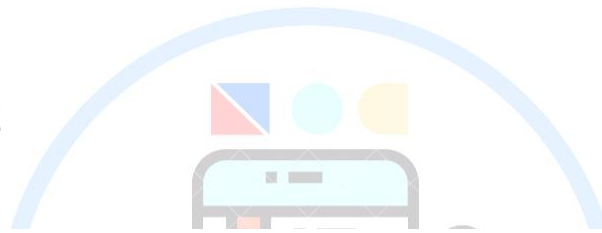
Marginal cost is referred to as the cost that is incurred by any business when there is a need for producing additional units of any goods or services.

It is calculated by taking the total cost of producing the additional goods into account and dividing that by the change in the total quantity of the goods produced.

Marginal cost includes variable costs like material and labour. It also includes increments in any fixed costs such as overhead, administrative, and selling.

The marginal cost formula is used to optimise the cash flow generation and is represented as follows:

Marginal cost = (Change in cost) / (Change in quantity)
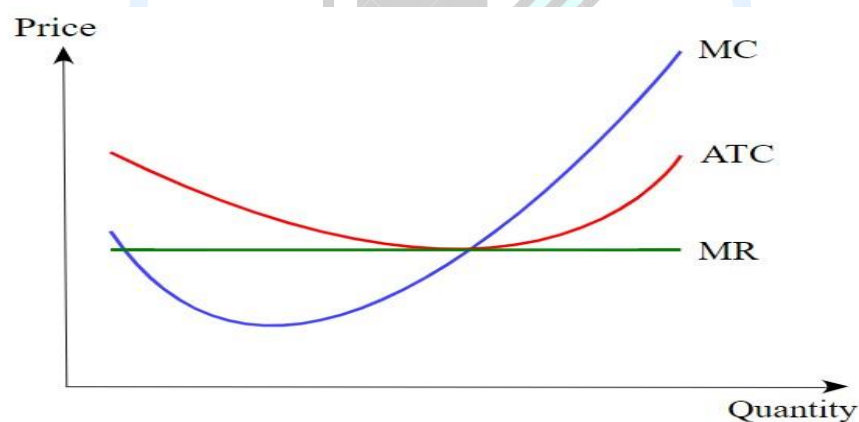
AVERAGE COST

TECHNIQUES OF ECONOMIC ANALYSIS PYQ's SOLUTION (ECB- 201 ,2021-22)

The average cost is the total cost divided by the number of goods produced. It is also equal to the sum of average variable costs and average fixed costs. Average cost can be influenced by the time period for production (increasing production may be expensive or impossible in the short run). Average costs are the driving factor of supply and demand within a market.

Average cost = Total cost of the units/Number of units

## Relationship Between Average and Marginal Cost

Cost curve: This graph is a cost curve that shows the average total cost, marginal cost, and marginal revenue. The curves show how each cost changes with an increase in product price and quantity produced.

- When the average cost declines, the marginal cost is less than the average cost.
- When the average cost increases, the marginal cost is greater than the average cost.
- When the average cost stays the same (is at a minimum or maximum), the marginal cost equals the average cost.

The following diagram illustrates the average-marginal cost relationship:

n this figure, A represents the average cost and M represents the marginal cost. It can be clearly seen that when marginal cost (M) is above the average cost (A), the average cost rises which is shown by the rising arrow. On the other hand, when the marginal cost (M) is below the average cost (A), then the average cost falls, as is shown by the falling arrow. But when the marginal cost is the same as the average cost (i.e., AM), the average cost remains constant, as if M is pulling A along horizontally.

Average-Marginal Relationship

NotesOcel
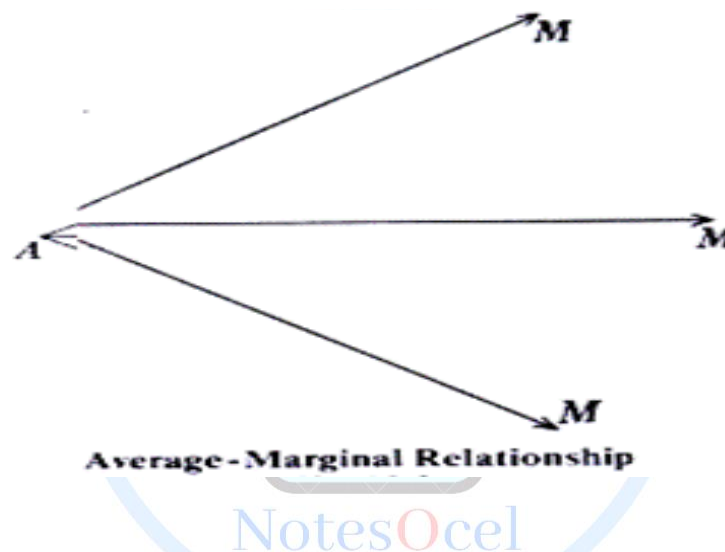
Q 14 Define Price elasticity of demand. Explain its various kinds with the help of suitable diagrams.

Refer Q 6

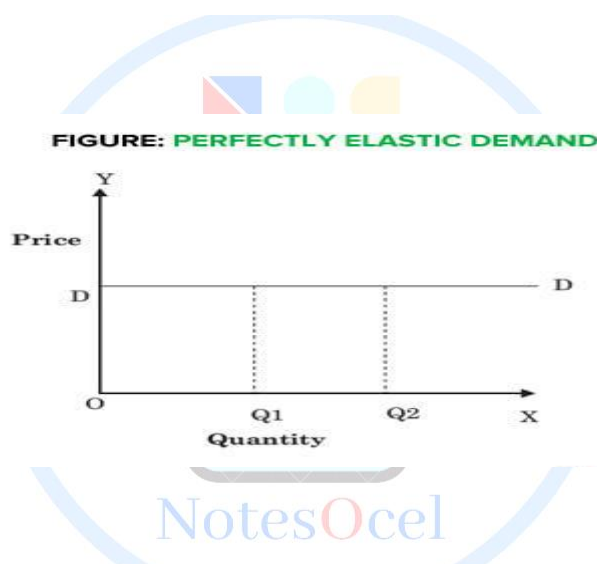**Different types of price elasticity of demand.**

Perfectly Elastic Demand

Perfectly Elastic Demand Definition: When a small change (rise or fall) in the price results in a large change (fall or rise) in the quantity demanded, it is known as **perfectly elastic demand**.

Under such type of elasticity of demand, a small rise in price results in a fall in demand to zero, while a small fall in price causes an increase in demand to infinity. In such a case, the demand is perfectly elastic or $e_p = \infty$.

Suppose product X is manufactured by a large number of sellers in the market. If a person wants to buy the product X, he could choose among different firms for the purchase. Let's say, firm A increased the price of product X, above market equilibrium. As a result, the demand for the product X for the firm would decrease to a great extent as the same product is available with other sellers too at cheaper prices. Thus, the demand for product X of the firm A is perfectly elastic.

FIGURE: PERFECTLY ELASTIC DEMAND

NotesOcel

Perfectly Inelastic Demand

Perfectly Inelastic Demand Definition: When a change (rise or fall) in the price of a product does not bring any change (fall or rise) in the quantity demanded, the demand is called **perfectly inelastic demand**.

In this case, the elasticity of demand is zero and represented as $e_p = 0$.

Graphically, perfectly inelastic demand curve is represented as a vertical straight line (parallel to Y-axis). Figure shows the perfectly inelastic demand curve.
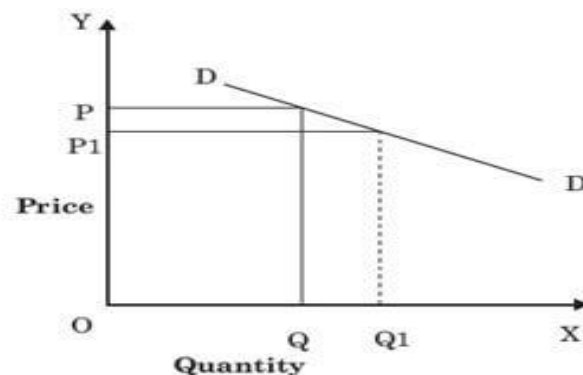
FIGURE: PERFECTLY INELASTIC DEMAND
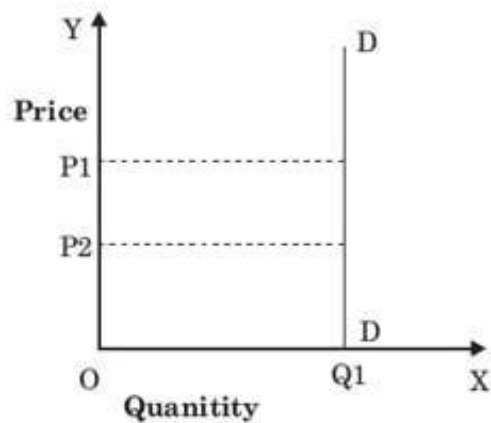
Relatively Elastic Demand

Relatively Elastic Demand Definition: When a proportionate or percentage change (fall or rise) in price results in greater than the proportionate or percentage change (rise or fall) in quantity demanded, the demand is said to be **relatively elastic demand**.

In other words, a change in demand is greater than the change in price. Therefore, in this case, elasticity of demand is greater than 1 and represented as **ep > 1**. The demand curve of relatively elastic demand is gradually sloping, which is shown in Figure.

**FIGURE: RELATIVELY ELASTIC DEMAND**

Relatively Inelastic Demand

Relatively Inelastic Demand Definition: When a percentage or proportionate change (fall or rise) in price results in less than the percentage or proportionate change (rise or fall) in demand, the demand is said to be **relatively inelastic demand**.

In other words, a change in demand is less than the change in price. Therefore, the elasticity of demand is less than 1 and represented as $e_p < 1$. The demand curve of relatively inelastic demand is rapidly sloping, which is shown in Figure.

FIGURE: RELATIVELY INELASTIC DEMAND
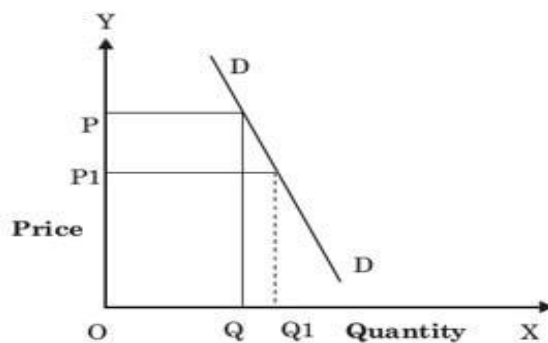
Unitary Elastic Demand

Unitary Elastic Demand Definition: Unitary elastic demand occurs when a change (rise or fall) in price results in equivalent change (fall or rise) in demand. The numerical value for unitary elastic demand is equal to one, i.e., $e_p = 1$. The demand curve for unitary elastic demand is a rectangular hyperbola, which is shown in Figure.

FIGURE: UNITARY ELASTIC DEMAND
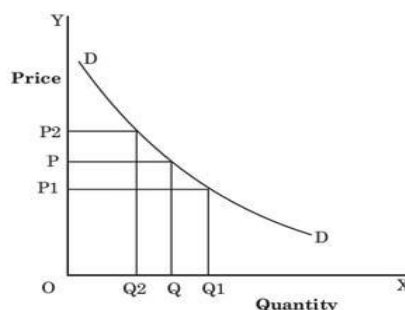
2017-2018

Q 15 What is rectangular hyperbola ?

Rectangular hyperbola is a curve under which all rectangular areas are equal. When the elasticity of demand is equal to unity (e_d = 1) at all points of demand curve, then the demand curve is rectangular hyperbola. It is a downward sloping curve as given in figure below.

FIGURE: UNITARY ELASTIC DEMAND
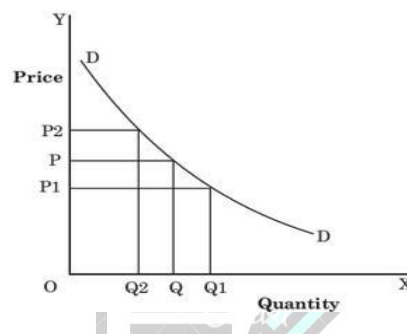
-> The general equation of a rectangular hyperbola is $x^2 - y^2 = a^2$ or $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
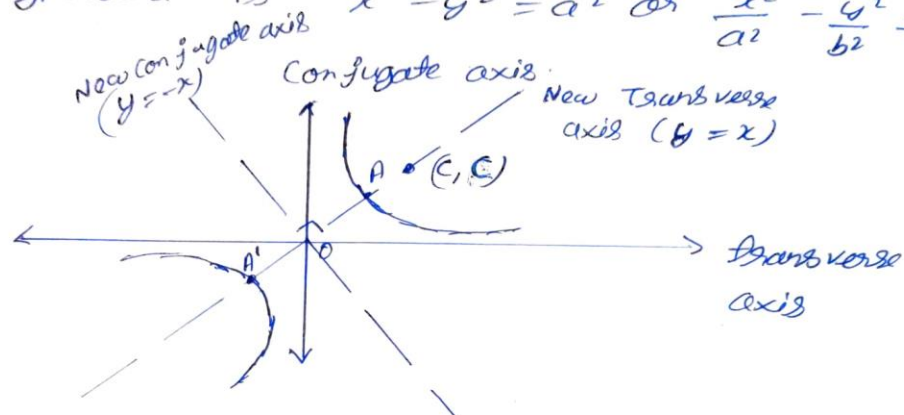
where a is length of semi major axis

i.e, $OA' = OA = a$

2018-2019

Q 16 What is implicit & explicit function. Distinguish them.

Implicit Function

A relationship between x and y is defined as an implicit function.
There is no separation of dependent and independent variables.
In other words, x and y are inseparable.
Example: x^2+2xy+y^2=0

Explicit Function

"An explicit function is one in which the dependent Variable and independent variable are separated by the equality on opposite sides.
The dependent variable is expressed in terms of the independent variable in this case.
To put it another way, x and y it can be separated.
y=x^2+x+1

Implicit Function	Explicit Function
An implicit function is one that has several variables, one of which is a function of the other set of variables.	An explicit function is one in which the dependent variable can be written explicitly in terms of the independent variable.

f(x, y) = 0 is the general form of an implicit function.

y = f(x) is the general form of an explicit function.

Example: x2– y2 = 0

Example: y = x + 4

2019-2020

Q 17 Explain the fundamentals problems of an economy. How they are solved by the modern welfare government.

Refer Q 10

How Market Mechanisms Solve the Basic Problems of an Economy?

All the three kind of economies, Capitalistic economy, Socialistic economy and Mixed economy, solve the basic problems of an economy in two methods:

- Free price mechanism
- Controlled price system which is also called State intervention

The Basic Problem of an Economy and Free Price Mechanism

A system of guiding the decisions of individuals within an economy through the price which is determined with the help of market forces of demand and supply is called price mechanism. This system is free of any government intervention. When the market equilibrium is reached by market forces of demand and supply, i.e. the quantity supplied becomes equal to the quantity demanded, then the price of a commodity is determined. Price mechanism also facilitates the determination of

resource allocation, consumption and production as well as determining the level of savings and factor income. This method mostly takes place in a capitalistic economy.

The Basic Problem of an Economy and State Intervention System

The government acquires the necessary productive resources for these activities and employ them in conformity with its priorities. The production pattern of the public sector, the prices of output items of the public sector and other measures are used to regulate the allocation of resources in private sector as well. These other measures include price controls, licensing, taxation, subsidies, and others. Additionally, various labour welfare measures are undertaken. Similar steps are taken to encourage the use d productive resources for encouraging the development of backward areas of the country, for removing specific shortages, and for bringing about a balanced development of the economy as a whole.

This system is defined by administering the fixed prices of every commodity. In a socialist economy, the government plays a vital role in determining the price of commodities. Ceiling price or floor price may be introduced by the government to regulate the prices of certain commodities.

The government takes various measures to improve agriculture and develop trade and industry. Eg. Grant of subsidies, loans at lower rate of interest, administered and support prices are announced for crops.

Q 18 Prove that rate of fall of MR is twice the rate of fall of AR

∴ any line is of form $y = mx + c$

↑ slope ↑ y-intercept

∴ AR is price also

∴ $P = mQ + c$

∵ $TR = P \times Q$

$TR = (mQ + c) Q$

$= mQ^2 + cQ$

also $MR = \dfrac{\Delta TR}{\Delta Q} = 2Qm + c$

Here $AR = P = \underline{mQ} + c$ & $MR = \underline{2Qm} + c$

∴ Here slope of MR is twice of AR, thus rate of change will be proportionate to slope.

Q 19 Distinguish between product and quotient rule of differentiation ?

Product Rule

Product rule in calculus is a method used to find the derivative of any function given in the form of a product obtained by the multiplication of any two differentiable functions. The product rule in words states that the derivative of a product of two differentiable functions is equal to the sum of the product of the second function with differentiation of the first function and the product of the first function with the differentiation of the second function. That means if we are given a function of the form: f(x)·g(x), we can find the derivative of this function using the product rule derivative as,

$$\frac{d}{dx}(f(x) . g(x)) = f'(x) . g(x) + f(x) . g'(x)$$

Quotient rule

A Quotient Rule is stated as the ratio of the quantity of the denominator times the derivative of the numerator function minus the numerator times the derivative of the denominator function to the square of the denominator function. In short, the quotient rule is a way of differentiating the division of functions or the quotients. This is also known as the **quotient rule differentiation** in maths.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}f(x) \cdot g(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$$

Q 20 Explain various properties of determinants giving suitable examples.

I. Concept Clarified

1. Determinant of a Square Matrix (up to 3 × 3 matrices)

The determinant is a scalar value that can be computed from the elements of a square matrix and has certain properties of the linear transformation described by the matrix. The determinant of a matrix A is denoted det(A), det A, or $|A|$.

➤ **First Order Determinant**

If matrix A = [a], then its determinant $|A|$ = a

➤ **Second Order Determinant**

A square matrix of order 2 × 2 gives determinant of second order. Its determinant can be obtained as follows :

Suppose we are given a matrix of order 2 × 2 then it's determinant is obtained by the following formula.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = ad - bc$$

➤ **Third Order Determinant**

A square matrix of order 3 × 3 gives determinant of third order.

Suppose, we are given a matrix of order 3 × 3 then its determinant is obtained by following way ;

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$|A| = a\begin{vmatrix} \bullet & \bullet & \bullet \\ \bullet & e & f \\ \bullet & h & i \end{vmatrix} - b\begin{vmatrix} \bullet & \bullet & \bullet \\ d & \bullet & f \\ g & \bullet & i \end{vmatrix} + c\begin{vmatrix} \bullet & \bullet & \bullet \\ d & e & \bullet \\ g & h & \bullet \end{vmatrix}$$
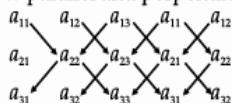
$$|A| = a(ei - hf) - b(di - gf) + c(dh - ge)$$

> **Evaluation of Determinant of Square Matrix of Order 3 by Sarrus Rule**

If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then determinant can be formed by enlarging the matrix by adjoining the first two columns

on the right and draw lines as shown below parallel and perpendicular to the diagonal.

The value of the determinant, this will be the sum of the product of element in line parallel to the diagonal minus the sum of the product of elements in line perpendicular to the line segment. Thus,

$\Delta = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$.

Note: This method doesn't work for determinants of order greater than 3.

> **Properties of Determinants**

1. The value of the determinant remains unchanged if both rows and columns are interchanged, *i.e.* $|A'| = |A|$.

2. If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes, *i.e.* det (A) becomes $[-$ det (A)$]$.

3. If any two rows (or columns) of a determinant are identical *i.e.* all corresponding elements are same, then the value of the determinant is zero, *i.e.* $|A| = 0$.

4. If each element of a row or column is zero then its determinant is zero. *i.e.* $|A| = 0$.

5. If each element of a row (or a column) of a determinant A is multiplied by a constant 'k', and new determinant obtained is determinant B, then determinant B will be 'k' times determinant A, *i.e.* $|B| = k |A|$.

6. If some or all elements of a row or column of a determinant are expressed as the sum of two (or more) terms, then the determinant can be expressed as the sum of two (or more) determinants.

   Example: $\begin{vmatrix} a+k & b+l & c+m \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} + \begin{vmatrix} k & l & m \\ d & e & f \\ g & h & i \end{vmatrix}$

7. If A is a matrix of order $n \times n$ then, $|kA| = k^n |A|$

8. The value of determinant remain same if we apply the operation $R_i \rightarrow R_i + k R_j$ or $C_i \rightarrow C_i + k C_j$

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